Enhancing heat transfer is of utmost importance in modern industrial applications. Pure liquids for illustration ethylene glycol, propylene glycol and water having lower conductivity are commonly used as cooling liquids in distinct applications. This approach helps conserve and optimize the enhancement of heat transportation. However, in order to achieve enhanced thermal efficiency, state-of-the-art liquids known as nanoliquids have been recommended. Thus the Buongiorno two-component nanoliquid model, which exhibits superior thermal efficiency compared to the aforementioned standard cooling liquids is being considered for formulating and analyzing the behavior of Casson nanoliquid configured by cylindrical convected surface. The problem formulation incorporates various factors such as Darcy-Forchheimer porosity, thermophoresis, magnetohydrodynamics, Brownian diffusion, suction/injection and Joule heating. Boundary-layer stretching flow is formulated. Dimensionless differential form from governing nonlinear problems is achieved by employing relevant variables. The application of the homotopy procedure results in convergent solutions for strongly nonlinear systems. The graphs are used to reveal the plots of significant factors in the analysis.

**Keywords**: buongiorno two-component model, darcy-forchheimer porosity, electro-magnetic characteristics, boundary-layer stretching flow, suction/injection.

### 1. Introduction

Nanotechnology refers to the field of science that focuses on comprehending the fundamental principles of materials science, biology, physics and chemistry at nanoscale [1]. The remarkable heat transportation performance exhibited by nanofluids [2, 3] containing nano-sized particles has garnered a significant consideration of modern researchers. A conventional heat-transferring fluid [4] can be transformed into a nanofluid by incorporating a lower concentration of nanowires, nanoparticles or nanotubes. The most frequently utilized types of elements are carbon-based nanoparticles, metal-oxide nanoparticles (including Al$_2$O$_3$, CuO, TiO$_2$) and metal nanoparticles (such as Au, Ag, Cu). Incorporating even a small quantity of solid particles into a nanofluid can significantly elevate its thermal conductivity [5] and enhance its heat transfer coefficient during laminar flow [5, 6] in comparison to base fluid. The remarkable thermal properties of nanofluids have numerous potential applications in various heat transfer industries, such as electronics, engine cooling, energy systems, transformer oil, solar heating and heat exchange mechanisms [7]. The exceptional heat transfer attributes of nanofluids are of significant interest to scientists, given their other remarkable characteristics. Modern researchers considered multi-physical assumptions to model fluid flow rheological problems based on nanofluids (see Refs. [8-16] and numerous investigations therein).

Scientists and engineers typically focus on working with fluids like oil, air and water which demonstrate...
Newtonian characteristics under diverse physical circumstances. In certain scenarios, relying solely on Newtonian models is unreliable and it becomes necessary to elaborate the fluids aspects elaborating non-Newtonian characteristics. Such situation frequently emerges in the industries of plastics manufacturing and chemical processing. Non-Newtonian characteristics can also be witnessed in mining industry, where the handling of muds along with slurries is a common occurrence. Additionally, this behavior is encountered in various other fields including pipeline industry, biomedical transport and tissue engineering. The industry places great importance on simulating non-Newtonian flows. The recent advancements in biofluid dynamics and polymer manufacturing have sparked a growing interest in understanding the flow featuring non-Newtonian rheology via tubes subjected to a constant yield value. Among these fluids, the Casson fluid is commonly used. Casson fluids are liquids that unveil shear thinning behavior, meaning their viscosity declines with higher shear rate. They have infinite viscosity at zero shear rate and possess zero yield stress and infinite shear rate. Currently, the Casson model is also being employed to develop rheological models for human blood. The literature (see Refs. [17-23]) provides detailed analyses of Casson fluid rheological models for non-Newtonian viscoplastic fluids. These fluids, the Casson fluid is commonly used. Casson fluids are liquids that unveil shear thinning behavior, meaning their viscosity declines with higher shear rate. They have infinite viscosity at zero shear rate and possess zero yield stress and infinite shear rate. Currently, the Casson model is also being employed to develop rheological models for human blood. The literature (see Refs. [17-23]) provides detailed analyses of Casson fluid rheological models for non-Newtonian viscoplastic fluids. These fluids, the Casson fluid is commonly used.

The current research focuses on examining the behavior of a convectively heated non-Newtonian viscoplastic (Casson) nanofluid subjected to thermal radiation. The analysis incorporates various factors such as Darcy-Forchheimer porosity, thermophoresis, magnetohydrodynamics, Brownian diffusion, suction/injection and Joule heating. Analytical procedure (homotopy analysis method [24-27]) is utilized to obtain convergent solutions for these highly nonlinear systems. The results are presented through graphs, revealing the influence of key parameters. Magneto-electric current [28], micro-channels transport [29], nano-based materials [30-32], heat transport [33, 34] are some recent and valuable research in flow of fluid versus different flow assumptions.

2. Modeling

Consider a viscoplastic nanofluid which accelerate subjected to convectively heated stretchable permeable cylinder. Rheological features of Casson fluid are described considering incompressibility and steady-state condition. The problem is represented using a cylindrical coordinate system and Fig. 1, provides a visual depiction of the geometry. Modeling features multiphysical aspects such as Darcy-Forchheimer porosity, thermophoresis, magnetohydrodynamics, Brownian diffusion, suction/injection and Joule heating. The aforesaid assumptions correspond to following expressions:

\[ \frac{\partial (\mu u_x)}{\partial x} + \frac{\partial (\mu u_r)}{\partial r} = 0, \]

\[ u \frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial r} = \nu \left[ 1 + \frac{1}{\beta} \left( \frac{\partial u_x}{\partial r} + \frac{1}{r} \frac{\partial u_x}{\partial r} \right) \right] \]

\[ -\frac{k}{\rho c_p} \left( \frac{\partial T}{\partial x} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{T_0}{T_\infty} \frac{\partial C}{\partial x} + \frac{16 \sigma \sigma_0^2}{3k' \rho c_p} \left( \frac{\partial T}{\partial r} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \]

\[ \frac{\partial C}{\partial x} + \frac{\partial C}{\partial r} = \frac{D_0}{T_\infty} \left[ \frac{\partial T}{\partial r} + \frac{1}{r} \frac{\partial T}{\partial r} \right] \]

\[ u = U_\infty \frac{\rho c_p}{1}, \quad v = -\nu_x, \quad -k \frac{\partial T}{\partial r} = h(T_r - T), \quad -D_0 \frac{\partial C}{\partial r} = h_c (C - C) \]

\[ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } r \rightarrow \infty \]

In the aforementioned Eqs. (1)-(5), velocity components \((u, v)\) are in \((x, r)\) directions respectively, \(I\) characteristic length, \(\nu = \frac{\mu}{\rho c_p}\) kinematic viscosity, \(\beta\) material parameter, \(\mu\) dynamic viscosity, \(B_0\) magnetic field potency, \(\tau = \frac{(\rho c_p)}{\rho c_p}\) heat capacity ratio, \(\rho_f\) liquid density, \((D_0, D_0)\) (thermo-poretic, Brownian) diffusion, \(\sigma\) electrical conductivity of liquid, \((\rho c_p)/(\rho c_p))\) (heat-capacity of liquid, nanoparticles effective heat-capacity), \(k'\) coefficient of mean-absorption, \((T_r, C_r)\) convective liquid (temperature, concentration), \(\sigma^2\) Stefan-Boltzmann constant, \((U_\infty, U_\infty, v_\infty)\) (stretching, reference, suction/injection) velocity, \((h_s, h_i)\) convective (mass, heat) transfer coefficients and \((T_\infty, C_\infty)\)
ambient liquid (temperature, concentration).

Employing

\[ \eta = \frac{U_r}{v} \left( r^2 - R^2 \right) / 2R, \quad \psi = \sqrt{U_r v R f(\eta)}, \quad \theta(\eta) = \frac{T - T_c}{T_f - T_c}, \]

\[ \phi(\eta) = \frac{C - C_a}{C_f - C_a}, \quad u = U_r f'(\eta), \quad v = \frac{\sqrt{U_r R}}{r} f(\eta), \]

the continuity equation (i.e., Eq. (1)) is satisfied trivially while Eqs. (2)-(6) yield:

\[ (1+2\gamma \eta) \left[ 1 + \frac{1}{\beta} \right] f'' + 2\gamma f' + f'' - \beta \left[ 1 + \frac{1}{\beta} \right] f = 0, \]

\[ -\beta_2 f'' + H_a f' = 0, \]  

\[ (1+2\gamma \eta) \left[ 1 + \frac{4}{3} R \right] \theta'' + 2\gamma \left[ 1 + \frac{4}{3} R \right] \theta' + Pr f \theta' \]

\[ + Pr(1+2\gamma \eta) \left( N_r \theta' + N_r \theta'' \right) + Pr E \Delta H_a f'' = 0, \]

\[ (1+2\gamma \eta) \phi'' + 2\gamma \phi' + Scf \phi' + \frac{N_r}{N_b} \left( 1 + 2\gamma \eta \right) \theta'' + 2\gamma \theta' = 0, \]  

\[ f = S, \quad f' = 1, \quad \theta'(0) = -\gamma_1 (1 - \theta(\eta)), \]

\[ \phi'(0) = -\gamma_2 (1 - \phi(\eta)) \quad \text{at} \quad \eta = 0, \]

\[ f'' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty. \]  

In Eqs. (7)-(10), \( \gamma = \frac{v^2}{\sqrt{wR}} \) illustrates curvature parameter, \( \beta_2 = \frac{\tau_\theta}{\tau_v} \) the inertia variable, \( Pr = \frac{m_r}{m_t} \) Prandtl number, \( \beta_2 = \frac{c_f}{c_v} \) Darcy porosity variable, \( N_r = \frac{c_D(T_r-T_c)}{\alpha_k} \) thermodiffusive factor, \( \tau_\theta = \frac{4\pi c_f}{c_v} \) radiation variable, \( N_b = \frac{\rho_0 c_D c_f}{\alpha_k} \) Brownian diffusion factor, \( S = \nu \frac{v^2}{\sqrt{wR}} \) (suction \( S > 0 \)) injection \( S < 0 \) parameter, \( \gamma_1 = \frac{k}{x_u \sqrt{c}} \) solutal, thermal Biot numbers, \( Ha = \frac{c_D^{\frac{1}{2}}}{c_v^{\frac{1}{2}}} \) Hartman number and \( Ec = \frac{v^2}{c_f(T_f-T_c)} \) Eckert number.

The mathematical forms of drag force \( (C_f) \) together with heat-mass transference rates \( (Nu_u, Sh_h) \) are expressed as:

\[ C_f = \frac{\tau_f}{U_r}, \quad \tau_f = \left[ \mu \left( 1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial r} \right]_{r=R}, \]  

\[ Nu_u = \frac{x u}{k(T_f-T_c)}, \quad q_u = -k \left( \frac{\partial T}{\partial r} \right)_{r=R} - 16\sigma T^4 \left( \frac{\partial T}{\partial r} \right)_{r=R}, \]

\[ Sh_h = \frac{x h}{D_h (C_f-C_a)}, \quad q_s = -D_h \left( \frac{\partial C}{\partial r} \right)_{r=R}. \]

Eqs. (11)-(13) in dimensionless forms yield:

\[ C_f, Re_r^{1/2} = \left( 1 + \frac{1}{\beta} \right) f'(0), \]

\[ Nu, Re_r^{1/2} = -\left( 1 + 4 \frac{3}{R} \right) \theta'(0), \]

\[ Sh_h, Re_r^{1/2} = -f'(0), \]  

in which Reynolds number is denoted by \( Re = \frac{IU}{v} \).

### 3. Computational Analytical Scheme

This section features analytical solutions obtained through homotopic algorithm [24-27]. This algorithm requires base-functions \( \left( f_0(\eta), \theta_0(\eta), \phi_0(\eta) \right) \) together with operators \( \left( L_f, L_\theta, L_\phi \right) \) in the forms given below:

\[ f_0(\eta) = S + \left( 1 - e^{-\gamma_1} \right), \]

\[ \theta_0(\eta) = \frac{\gamma_1 \exp(\gamma_2)}{\gamma_1 + \gamma_2}, \]

\[ \phi_0(\eta) = \frac{\gamma_2 \exp(\gamma_2)}{\gamma_1 + \gamma_2}, \]

\[ L_f = f'' - f', \]

\[ L_\theta = \theta'' - \theta, \]

\[ L_\phi = \phi'' - \phi. \]

The operators mentioned above obey:

\[ L_f(C_1 + C_2 e^{\sigma} + C_3 e^{-\eta}) = 0, \]

\[ L_\theta(C_1 e^{\sigma} + C_2 e^{-\eta}) = 0, \]

\[ L_\phi(C_1 e^{\sigma} + C_2 e^{-\eta}) = 0, \]

where \( C_i (i = 1-9) \) signify arbitrary constants.

### 4. Convergence

No doubt the utilization of homotopic algorithm yields series solutions which may converge or diverge. One has to ensure the convergence of obtained results while using homotopic algorithm. To achieve this, an assisting factor \( (h) \) is utilized to adjust and monitor the convergence region of obtained series solutions. By analyzing the \( h \)-curves depicted in Fig. 2, optimized estimations for \( h_f, h_\theta, \) and \( h_\phi \) are \(-1.25 < h_f < -0.30, \ -1.8 < h_\theta < -0.25 \) and \(-1.65 < h_\phi < -0.26 \) respectively. These ranges of \( h_f, h_\theta, \) and \( h_\phi \) are attained at 12th order approximations. Numerical analysis, as shown in Table 1, also confirms the convergence arithmetically. It is evident that Eqs. (7)-(9) converge at 13th order approximations. Besides the
Parametric values are selected arbitrarily as \( \gamma = N_t = \beta_1 = N_b = \beta_2 = 0.1, \; Ha = R = 0.2, \; S = \gamma_2 = Ec = 0.3, \; \gamma_1 = 0.4, \; Pr = Sc = 1.1 \) and \( \beta = 1.5 \).

Figs. 2 and 3 elaborate the temperature response when nanoscale factors, specifically thermophoresis \( (N_t) \) and Brownian movement \( (N_b) \) are varied independently. A noticeable increase in temperature is found subjected to increasing \( N_t \) and \( N_b \). The expression of thermophoretic body force (i.e., \( N_t \theta^2 \)) occurring in Eq. (8) is strengthened when the thermophoresis parameter \( N_t \) increases. The presence of a higher \( N_t \) value (see Fig. 2a) assists in the movement of nanoparticles in the presence of a larger temperature gradient. This leads to an elevation in the thermophoretic term, which in turn, upsurges the viscosity of the thermal boundary-layer on cylindrical sensor. Besides, from Fig. 2b, it is scrutinized that higher \( N_b \) estimations corresponds to nanoparticles with smaller diameters. This leads to more frequent and energetic collisions between the nanoparticles, resulting in intensified chaotic motions and subsequently elevated temperatures.

Larger \( N_b \) signifies that the Brownian motion factor \( (N_b) \) appearing in Eq. (8) through the term \( N_b \theta \phi' \) is amplified, resulting in an energized regime. This enhancement in Brownian motion factor \( (N_b) \) promotes heat transfers by increasing the thickness of thermal boundary-layer. Consequently, this confirms the heat enhancement aspects achieved through the utilization of nanofluids. Fig. 3a demonstrates that increasing Eckert number \( (Ec) \) corresponds to higher temperature. This augmentation indicates the
presence of viscous heating effects within the nano-polymer. The importance of viscous heating should not be ignored, predominantly considering its impact in dynamics of polymer coatings. The Eckert number $Ec = \frac{U^2}{\kappa (r - r_0)}$ represents the relation of kinetic energy dissipated in the flow to enthalpy difference in the boundary-layer. Note that $Ec > 0$, heat transfers away from cylindrical surface towards the boundary-layer. Neglecting this effect in thermally magnetized nano-coating based models would result in underestimating temperatures in the boundary layer and less accurate characterization of thermal distribution. Analysis of Fig. 3b indicates a substantial increase in temperature when $\gamma_1$ upsurges. As $\gamma_1$ upsurges, there is an enhanced heat transportation from wall towards boundary-layer, which is a progressive effect. This encourages thermo diffusion in the nano-polymer and leads to a heating aspect, resulting in a thicker thermal boundary-layer.

Fig. 4a reveals that higher $N_t$ counteracts the impact of Brownian movement ($N_b$). There is a significant intensification in nano-particle concentrations, resulting in an elevated thickness of species boundary-layer. Clearly $N_t$ and $N_b$ occur in energy Eq. (8) via terms $N_t \theta' \phi' + N_t \theta'^2$ while they also emerge in Eq. (9) through the term $\frac{N_t}{N_c} \left(1 + 2\gamma \eta \right) \theta'$. Thermophoresis promotes the migration of particles from hotter to colder zones and aids in species diffusion, leading to an increase in the thickness of concentration boundary-layer (see Fig. 4a). On the other hand, Brownian movement disperses nanoparticle distribution because of chaotic motion resulting from flow

**Fig. 4.** (Color online) (a) $N_t$ variations against concentration, (b) $N_b$ variations against concentration, $\gamma_2$ variations against concentration.

**Fig. 5.** (Color online) (a) $N_t$ and $N_b$ variations against Nusselt number, (b) $\gamma_2$ and $Sc$ variations against Sherwood number.
acceleration, thereby reducing the thickness of concentration boundary-layer (see Fig. 4b). The increase in solutal Biot number ($\gamma_2$), as computed in Fig. 4c has a pronounced effect of elevating nanoparticles concentration ($\phi(\eta)$), where $\phi(\eta)$ is an increasing function of $\gamma_2$. As a result, there is a substantial enhancement in the thickness of concentration boundary-layer accompanying the escalation in $\gamma_2$.

The heat transportation rate (i.e., local Nusselt number ($Nu_{xx}$)) is reduced with an increase in $N_t$ and $N_b$ since these nanoscale effects aid in heating boundary-layer and diminish thermal transportation to wall (see Fig. 5a). Consequently, heat transportation rates can be effectively controlled by precisely selecting these thermophysical factors. On the other hand, a reverse situation is witnessed against mass transportation rate (i.e., local Sherwood number ($Sh_{xx}$)) when $\gamma_2$ and $Sc$ are augmented.

5. Conclusions

A mathematical and theoretical study featuring magnetization impact in viscoplastic non-Newtonian Casson model utilizing two-component Buongiorno's model is presented. The analysis incorporates various factors such as Darcy-Forchheimer porosity, thermophoresis, magnetohydrodynamics, Brownian diffusion, suction/injection and Joule heating. This simulation aimed to simulate the processing of higher temperature based nano-polymeric materials. The main outcomes extracted from the presented simulations are stated below:

- Increasing thermophoresis ($N_t$) and Brownian movement ($N_b$) tend to upsurge the nanofluid temperature distribution. On the other hand, heat transportation rate is reduced with an increase in $N_t$ and $N_b$.
- Enlarging Eckert number and thermal Biot number enhance temperature.
- Nanoparticles species concentration is significantly reduced for increasing estimations of Brownian motion while it discloses reverse influence nanoparticles species concentration for thermophoresis factor.
- Enhancing estimations of solutal Biot number significantly upsurge the nanoparticles concentration and mass transportation.

The elaborated research has shown the impressive effectiveness of HAM in simulating the dynamic flow of nonlinear thermally magnetized nano-polymeric coatings. However, the focus has been primarily on the Darcy-Forchheimer porosity model. In future researches, more sophisticated non-Newtonian models featuring accurate Darcy-Forchheimer porosity term will be explored, and the findings will be communicated soon.

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