Darcy-Forchheimer Flow of Magnetized Bioconvective Williamson Nanofluid with Variable Thermal Conductivity

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In this article modeling and theoretical analysis of magnetized Williamson nanomaterial flow by permeable surface of cylinder is studied. The idea of self-propelled gyrotactic microorganisms is implemented to stabilize the suspended nanoparticles in Williamson liquid. Darcy-Forchheimer together with porosity effects are accounted in the flow. Energy relation is modeled in view of thermal radiation, variable thermal conductivity and Joule heating. Activation energy linked with chemical reaction is executed at the surface. Furthermore, Brownian dispersion and thermophoresis effects are also considered. Flow governing dimensional model is acquired using boundary layer suppositions. Suitable transformations are used to alter the system of PDE’s into non-dimensional. NDSolve code in Mathematica package is utilized to solve the model. Impacts of various flow regulating variables on velocity, temperature, mass concentration and motile density are investigated by plotting. Coefficient of skin friction, Sherwood number, motile density number and heat transfer rate are tabulated and analyzed. It is observed that velocity field decays while temperature field enhances versus rising dimensionless magnetic parameter. Moreover, due to magnetic field more Lorentz force is applied to the flow as a result surface drag force enhances while heat transfer rate decays.

Keywords: bioconvection, williamson nanofluid, activation energy, darcy-forchheimer, variable thermal conductivity

Nomenclature

\[(u, v)\] : velocity components
\[\Gamma_a\] : Williamson fluid constant
\[u_0\] : stretching rate constant
\[\rho\] : density of fluid
\[\mu\] : dynamic viscosity
\[\sigma\] : electrical conductivity
\[C_p\] : specific heat
\[\Delta \sigma\] : Schmidt number
\[T\] : temperature of nanofluid
\[\phi\] : dimensionless nanofluid concentration
\[\tau\] : heat capacity ratio
\[\theta\] : dimensionless temperature
\[f\] : dimensionless velocity
\[(r, x)\] : coordinates of cylinder
\[B_0\] : magnitude of magnetic field
\[C\] : nanofluid concentration
\[q_m\] : local mass flux
\[\alpha_d\] : variable thermal diffusivity
\[F\] : inertia coefficient
\[Sc\] : Schmidt number
\[b\] : chemotaxis constant
\[q_r\] : radiative heat flux
\[D_T\] : thermophoretic dispersion coefficient
\[E_a\] : activation energy
\[F_r\] : Forchheimer parameter
\[C_b\] : drag coefficient
\[Ec\] : Eckert number
\[K^+\] : Curvature variable

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1. Introduction

The notion of nanofluids was first introduced by Choi [1] after working observational investigations on different nanoparticles suspensions into carrier fluids. Nanofluids can be created by suspending nano sized particles like metallic oxide, metals, nitrides, metal carbides and carbon nanotubes into working fluids such as oils, water, glycol and ethylene. Thermophysical properties of carrier liquids enhances with the insertion of nanoparticles. Nanofluids have vital applications in various fields of science and engineering like mechanical cooling, disease treatment, indicative tests, chemical processes, heat exchangers and atomic reactors micro scale fluidic and several others. [2].

Bang and Chang [3] investigated heat transfer enhancement in water based nanofluid past over plain surface in fresh water. Hayat et al. [4] conducted a comparative study on MHD convective flow and heat transfer in Silver-water and Copper-water nanoliquid. Impact of activation energy, Joule heating and dissipation on slip flow of Prandtl-Eyring nanofluid is studied by Qayyum et al. [5]. Some valuable studies on nanofluids are listed through references [6-11].

Flow behavior of nanofluid under the effect of magnetic field is characterized as magnetohydrodynamics (MHD). MHD has significant practical applications like cooling of nuclear reactors and MHD generators, cancer therapy, study of plasma, exploration of oil, production of crystal fiber, production of paper, extraction of geothermal energy, controlling of boundary layer flows. Kumaran et al. [12] reported chemically reactive MHD flow of Maxwell and Casson nanoliquids with heat source/sink. Influence of Cattaneo-Christov diffusion and heat transfer performance in viscoelastic convective MHD flow with velocity slip boundary constraints is explored by Li et al. [13]. Unsteady 2-D flow behavior of MHD nanoliquid flow by impulsively rotating permeable disk is reported by Turkyilmazoglu [14, 15]. Hayat et al. [16] examined the boundary slip mechanisms in MHD flow of chemically reactive nanomaterial with dissipation. Waqas et al. [17] scrutinized the MHD micropolar nanofluid flow caused by stretched sheet with Joule heating, dissipation and convection taking convective boundary. Characteristics of Hall effects and variable viscosity on MHD flow in a duct with heat transfer analysis is inspected by Evcin et al. [18]. Hayat et al. [19] explored the features of irreversibility in MHD flow of nanoliquid caused by circular curved stretched sheet. Anuar et al. [20] inspected MHD carbon nanotube flow over nonlinearly deforming sheet with stability analysis. Khan et al. [21] inspected the melting effects on MHD incompressible viscous flow of nanoliquid by a stretched surface. Many researchers [22-27] have deliberated MHD flow problems in several flow fields.

Darcy’s Law is an expression that elucidates the fluid flow through a porous medium. This law is valid in the situations where less porosity and minimum velocity involve. Forchheimer manifested the factor of square of velocity in the equation of Darcy’s velocity to analysis the characteristics of inertia and boundary. Muskat and Meres...
[28] named this term as Forchheimer term. Seddeek [29] investigated mixed convection Darcy-Forchheimer flow under the influence of thermophoresis and dissipation. Significance of compressibility and uniform porosity on heat transfer performance in fluid flow by rotating disk is reported by Turkylmazoglu [30]. Pal and Mondal [31] promoted the theory of Darcy Forchheimer to investigate the hydromagnetic flow subject to variable viscosity and nonuniform heat source/sink in porous surface. Behavior of magnetized Cross nanofluid flow past over porous stretched surface with Darcy Forchheimer and improved homogeneous-heterogeneous reaction is revealed by Hayat et al. [32]. Modeling and numerical simulation for convective radiative flow with Darcy-Forchheimer and second order velocity slip is presented by Khan and Alzahrani [33]. Turkylmazoglu [34] studied characteristics of time dependent magnetized flow by rotating permeable disk with variable viscosity. Khan et al. [35] reported Darcy-Forchheimer flow of CNT’s based nanoliquid flow occupied between two stretched rotating disks.

Activation energy is the energy that must be provided to mixtures to result in a chemical reaction. This notion was first familiarized by Arrhenius in 1889. The Arrhenius relation offers the quantitative foundation for the correlation among the frequency at which a reaction and activation energy take place. Investigation of heat and mass transfer under the influence of activation energy and binary chemical reaction motivated numerous investigators on account of its vigorous practical usages in different fields like oil suspension, hydrodynamics, industries of oil storage and in geothermal sciences. Impacts of Arrhenius energy on mass transfer rates is scrutinized by Kalaivanan [36]. Influence of Arrhenius activation energy and thermal radiation in second grade nanoliquid with heat & mass transfer is considered by Kalaiyan and et al. [37]. Kumar et al. [38] explored the simultaneous impact of activation energy and magnetic field on nanoliquid flow over rotating disk. Yesodha et al. [39] examined chemically reactive flow in presence of activation energy, Soret and Dufour possessions. Theoretical examination of unsteady nanoliquid flow by stretched surface having convective boundary constraints is reported by Dhlamini et al. [40]. Impact of activation energy in non-Newtonian nanofluids over various stretching geometries is listed in references [41-44].

Keeping different engineering and industrial applications of Williamson nanofluid in mind, the prime objective of current study is to analyze the natural bio-convective flow of Williamson nanofluid by stretching cylinder with reflecting body forces such as Darcy Forchheimer, porous medium, thermal radiation and gyro tactic microorganisms. Simultaneous effects of buoyancy forces of nanoparticles and self-propelled gyro tactic microorganisms are accounted. Furthermore, chemical reaction with Arrhenius energy is also assimilated in formation of concentration relation. Influence of involved physical parameters on velocity, temperature, concentration and motile density of microorganism are discussed.

2. Mathematical Formulation

Here temperature dependent variable thermal conductivity in unsteady incompressible flow of Williamson nanofluid by permeable stretching cylinder is studied. Effects of magnetic field and Darcy-Forchheimer are accounted. Energy and concentration relations are respectively modeled in view of thermal radiation and Arrhenius energy related with chemical reaction. Furthermore, Brownian dispersion and thermophoresis effects on flow are considered. The cylinder is stretched with velocity \( u(x) = \frac{w}{x} \), due to which flow is generated (See Fig. 1). The concept of self-propelled gyro tactic microorganisms is executed to stabilize the suspended nanoparticles in Williamson nanofluid. In view of above assumptions the governing boundary layer flow model is expressed as [34, 44].

\[
\frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial u}{\partial r} = 0,
\]

\[
\rho u \frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial u}{\partial r} = \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial y^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial z^2} \right),
\]

\[
\frac{\partial v}{\partial x} + \frac{v}{r} \frac{\partial v}{\partial r} = -\frac{\sigma B_i u}{\rho C_p} - \frac{1}{K} \frac{\partial u}{\partial r} - F u^{\prime} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\sigma B_i^2}{\rho C_p} u^2,
\]

\[
\frac{\partial c}{\partial x} + \frac{v}{r} \frac{\partial c}{\partial r} = D \left[ \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial c}{\partial r} \right] + \frac{D}{r} \left[ \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial c}{\partial r} \right] + K_c \left( C - C_\infty \right) \left( \frac{T}{T_\infty} \right)^n \exp \left( \frac{-E_a}{RT} \right),
\]

\[
\frac{\partial n}{\partial x} + \frac{v}{r} \frac{\partial n}{\partial r} + \frac{h W c}{r} \frac{\partial c}{\partial r} + N \frac{\partial c}{\partial r} + N \frac{\partial c}{\partial r} + N \frac{\partial^2 c}{\partial r^2} + N \frac{\partial^2 c}{\partial r^2} + N \frac{\partial^2 c}{\partial r^2} = D \left( \frac{1}{r} \frac{\partial n}{\partial r} + \frac{\partial c^2}{\partial r} \right)
\]

\[
\frac{\partial c}{\partial x} + \frac{v}{r} \frac{\partial c}{\partial r} = D_n \left( \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial c^2}{\partial r^2} \right)
\]
Relation for linearly temperature dependent variable thermal conductivity is
\[ \alpha_a = \alpha_c (1 + \varepsilon \theta) , \] (7)

Using (7), (2) becomes
\[ u_T + v_T r + \frac{\partial T}{\partial r} \left[ D_a \frac{\partial T}{\partial r} + \frac{D_c}{\tau_a} \left( \frac{\partial T}{\partial r} \right)^2 \right] + \frac{1}{\rho C_p} \frac{\partial q}{\partial r} + \frac{\sigma B^2}{\rho C_p} u^2 = 0 . \] (8)

Letting
\[ \xi = \sqrt{\frac{\mu C_p}{2 \rho}} , \quad \phi(x) = \frac{C}{C_c}, \quad \frac{\partial T}{\partial r} = \frac{T_r - T_c}{T_c} . \] (9)

Condition (1) is satisfied while (2, 4-6, 8) take the form:
\[ f'f'' + 1 + 2K^2 \xi f'' + 2K^2 f'' + \frac{1}{2} (1 + 2K^2 \xi) \phi^2 + 2K^2 \phi' + Fr f'' = 0 , \] (10)

\[ \psi' + 1 + 2K^2 \xi \phi' + 2K^2 \phi' + \frac{\alpha_t}{\alpha_c} \phi' + 2K^2 \phi' + \frac{\alpha_t}{\alpha_c} \phi' + 2K^2 \phi' + \frac{\alpha_t}{\alpha_c} \phi' = 0 , \] (12)

with
\[ f(0) = 0 , \quad f'(0) = 1 , \quad \phi(0) = 1 , \quad \psi(0) = 1 , \quad \phi(0) = 1 , \quad \psi(0) = 0 , \quad \psi(0) = 0 , \quad \phi(0) = 1 , \quad \psi(0) = 0 . \] (13)

Mathematical form of non dimensional variables
\[ \frac{r^2}{R^2} = 1 + 2K^2 \xi , \quad \xi = \sqrt{\frac{\mu C_p}{2 \rho}} , \quad \phi(x) = \frac{C}{C_c}, \quad \frac{\partial T}{\partial r} = \frac{T_r - T_c}{T_c} . \] (15)

2.1. Engineering quantities
Surface drag force \( C_f \), heat transfer rate \( N_u \) mass transfer rate \( Sh \) and density number \( N_n \) are mathematically expressed as
\[ C_f = \frac{2 \tau_s}{\rho u_{\infty}^2} , \quad N_u = \frac{\lambda}{\alpha_c (T_r - T_c)} , \quad Sh = \frac{\lambda}{\rho C_p (T_r - T_c)} \] (16)

where
\[ \tau_s = \frac{\mu C_p}{2 \rho} \] (17)

Non dimensional forms are
\[ \frac{1}{2} C_f \tau_s^2 = f'(0) + \frac{1}{2} f''(0) , \quad N_u Re_t = -\phi'(0) , \quad Sh = -\phi'(0) , \quad N_n Re_t = -\psi'(0) \] (18)

Where \( Re_t = \frac{u_{\infty}^2}{v} \) represent local Reynolds number.

3. Solution Methodology
Nonlinear boundary value problem (10-13) along with boundary constrains (14) is locally similar for which numerical solution is computed by NDSolve code of Mathematica package. Computational results are acquired.
4. Solution Validation

After effectively simulating the dimensionless model in preceding section, this section is arranged to check the computed result. On this end, current results have been tested by comparing with the already published results of Abdelmalek [45] in Table 1. A good agreement of both the results is observed among the studies.

5. Results and Discussions

Physical impacts of flow influential variables on heat, mass, motile density transfer and surface drag force is analyzed numerically via tables. Variation in fluid velocity, volume fraction of microorganisms, temperature and mass concentration is scrutinized by various curves.

5.1. Velocity

In this subsection infiltration in velocity field ($f'(\xi)$) versus curvature variable ($K^o$), porosity parameter ($K_1$), inertial forces variable ($Fr$), Hartmann number ($Ha$) and Weissenberg number ($\Omega$) is examined. Fig. 2 depicts the variation in $f'(\xi)$ versus rising $K^o$. Here, we observe that velocity decelerates via higher $K^o$. Physically, when curvatures increases radius of cylinder reduces, as a result fluid contact area decays. Consequently more resistive force is applied on fluid flow and fluid velocity near the surface declines. From Fig. 3, one can observe that velocity field diminished for higher $K_1$. In fact, increment in $K_1$ offers more resistance in the flow path and that resistance is responsible to diminish the velocity field and layer thickness. Behavior of $f'(\xi)$ in view of $Ha$ is studied in Fig. 4. It is noticed from Fig. 4 that layer thickness and curves of velocity decelerates when $Ha$ takes higher values. Since higher $Ha$ offers more Lorentz force which is basically a resistive, due to which velocity field reduces.

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>Abdelmalek [45]</th>
<th>Present results</th>
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</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.4716</td>
<td>0.4717</td>
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</table>

Taking $Ha=0.12=Lb=\delta=0.1, \ K_1=Kb=Nh=0.3, \ Fr=Fr=\alpha=0.2, \ Pr=1.2, \ \epsilon=R=K^o=Ec=Pe=0.5, \ Sc=0.7, \ E_l=\sigma=1.0, \ \alpha_\infty=0.2, \ \text{and} \ n_l=2.0$. 

![Fig. 2. (Color online) Curves of $f'(\xi)$ via $K^o$.](image)

![Fig. 3. (Color online) Curves of $f'(\xi)$ via $K_1$.](image)

![Fig. 4. (Color online) Curves of $f'(\xi)$ via $Ha$.](image)

![Fig. 5. (Color online) Curves of $f'(\xi)$ via $Fr$.](image)
Fig. 5 is delineated to study the effect of $Fr$ on $f'(\xi)$. Clearly $f'(\xi)$ and associated layer thickness decays via higher $Fr$. Fig. 6 shows the variation in $f'(\xi)$ versus rising Weissenberg number, here we observe that velocity diminished for larger $\Omega$. Physically, higher $\Omega$ relaxation time improves, therefore $f'(\xi)$ and layer thickness declines.

5.2. Temperature

Impacts of Prandtl number ($Pr$), curvature variable $K^o$, thermophoresis parameter ($Nt$), radiation variable ($R$), Brownian dispersion parameter ($Nb$), Hartmann number ($Ha$) and thermal conductivity parameter ($\varepsilon$) on temperature ($\theta(\xi)$) is depicted in Figs. (7-12). Effect of $Pr$ on $\theta(\xi)$ is displayed in Fig. 7, here decreasing trend in $\theta(\xi)$ is observed for larger $Pr$. Since for higher $Pr$, the thermal diffusivity of nanoparticles reduces, consequently $\theta(\xi)$ and layer thickness diminished. From Fig. 8, we can see that fluid temperature and thermal layer thickness decays for larger $K^o$. Physically, higher $K^o$ fluid contact area reduces, as $K^o$ and radius of cylinder are in inverse relation, thus $\theta(\xi)$ retards when $K^o$ upsurges. Characteristics of $Nt$ and $Nb$ on $\theta(\xi)$ is discussed in Figs. 9 & 10. From these figures we examined that temperature of nanoparticles accelerates when $Nt$ and $Nb$ takes higher
estimations. Temperature field enhances via higher $R$ (see Fig. 11). Since rate of heat transfer to Williamson fluid increases as $R$ upsurges, thus $\theta(\zeta)$ and layer thickness enhances. Behavior of $Ha$ on $\theta(\zeta)$ is exposed through Fig. 12. Here, $\theta(\zeta)$ boost up via rising $Ha$. Increasing values of $Ha$ offer extra resistance to the system and as a consequences random motion of nanoparticles increases. Therefore $\theta(\zeta)$ and relevant layer improves for higher estimations of $Ha$. Fluid temperature falls down for rising $\varepsilon$ (see Fig. 13), it is due to the fact that for higher $\varepsilon$, bonds break with in liquid and the molecules free to travel. Thus higher $\varepsilon$ decays the $\theta(\zeta)$ and layer thickness.

5.3. Concentration

Figs. (14-20) are outlined to examined physical impact of chemical reaction variable ($\sigma$), activation energy parameter ($E_1$), curvature parameter ($K^*$), thermophoresis variable ($Nt$), Brownian diffusion parameter ($Nb$) and Schmidt number ($Sc$) on mass concentration $\phi(\zeta)$. Fig. 14 shows behavior of $\sigma$ on $\phi(\zeta)$, here higher $\sigma$ decays $\phi(\zeta)$ and layer thickness. Since for higher $\sigma$, rate of destructive chemical reaction increases which liquefies the species in fluid more rapidly. Therefore $\phi(\zeta)$ and layer thickness retards. Mass concentration inflates for higher estimations
of activation energy variable (see Fig. 15). Higher $E_1$ shrinks the improved Arrhenius function which eventually favors destructive reaction and thus mass concentration and layer thickness upsurges. The influence of curvature variable on concentration distribution is depicted in Fig. 16. Here, one can notice that $\phi(\xi)$ and associated layer thickness decays for rising $K^o$. Fig. 17 elucidates the characteristics of $Sc$ on $\phi(\xi)$. In fact kinematic viscosity improves for rising $Sc$ and thus $\phi(\xi)$ diminished. Physical impact of temperature difference ratio variable on $\phi(\xi)$ is sketched in Fig. 18. Fig. 18 is evident that $\phi(\xi)$ retards for higher approximations of $\delta$. Figs. 19 and 20 are outlined to explore the behavior of $\phi(\xi)$ versus rising $Nb$ and $Nt$. Here, we observe that $\phi(\xi)$ shows opposite trend via higher $Nb$ and $Nt$.

### 5.4. Motile Density

Influence of bioconvection Peclet number ($Pe$), bioconvection Lewis number ($Lb$), curvature parameter ($K^o$) and microorganisms concentration difference variable ($\Omega_a$) on motile density ($\varphi(\xi)$) is scrutinized through Figs. (21-24). From Fig. 21, it is noticed that motile density of gyrotactic microorganisms decays for lager estimations of $Pe$.
Since $Pe$ is associated with diffusivity of microorganism, so higher $Pe$ decays microorganisms, thus $\varphi(\zeta)$ diminishes. Physical impact of $Lb$ is depicted in Fig. 22. Here decreasing trend is observed in motile density and associated layer. Behavior of $\varphi(\zeta)$ in response of rising curvature variable is captured in Fig. 23. Here one can clearly visualize that $\varphi(\zeta)$ is decreasing function of $K_0$.

Fig. 24 is outlined to explore the performance of $\varphi(\zeta)$ versus increasing $\Omega_a$, it is examined that motile density curves retards via $\Omega_a$. Since improvement in $\Omega_a$ upsurges the microorganism concentration in ambient fluid due to which motile density decreases.

### 5.5. Engineering quantities

This slice concerns with the numerical investigation of essential engineering quantities like coefficient of skin friction, heat, mass and motile density transfer rates via involved variables. It is examined from Table 2 that magnitude of $C_f$ improves via higher estimations of $K_1$, $K_0$, $Ha$ and $Fr$ whereas negative impact of fluid material parameter is observed. Table 3 is organized to analyze the

### Table 2. Numerical results of skin friction ($C_f$) in view of $K_1$, $K_0$, $Ha$, $Fr$ and $\Omega$.

<table>
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<th>$K_1$</th>
<th>$K_0$</th>
<th>$Ha$</th>
<th>$Fr$</th>
<th>$\Omega$</th>
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### Table 3. Numerical results of heat transfer rate ($Nu_a$) in view of $K_0$, $Ha$, $Nb$, $Nt$, $Ec$, $Pr$, $R$ and $\varepsilon$.

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<tr>
<th>$K_0$</th>
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<th>$Nb$</th>
<th>$Nt$</th>
<th>$Ec$</th>
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<th>$R$</th>
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### Table 4. Numerical results of mass transfer rate ($Sh_a$) in view of $\delta$, $Nt$, $Nb$, $Sc$, $n_1$, $E_1$, $K_0$ and $\sigma$.

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### Table 5. Numerical results of density number ($Nn_x$) in view of $K_0$, $\Omega_a$, $Lb$ and $Pe$.

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variation in heat transfer rate \((Nu)\) versus selected values of \(K^o, Ha, Nt, Nb, Ec, Pr, R\) and \(\varepsilon\). Here improvement in intensity of Nusselt number is observed for higher Prand \(\varepsilon\) while reverse impact of \(Nb, Nt, Ec\) and \(R\) is noticed. Table 4 demonstrates that mass transfer rate upsurges for higher estimations of \(\delta, Nt, Nb, n_1, E_1, \sigma\) and \(K^o\). however it retards against \(E_1\). Table 5 explores that density number is decreasing function of \(\Omega_a, Lb, Pe\) and \(K^o\).

6. Final Remarks

Key observations of current investigation are:

- Velocity of nanoliquid is decreasing function of \(K^o, K, Ha, Fr\) and \(\Omega\).
- Temperature field upsurges for higher estimations of \(Nt, Nb, R\) and \(Ha\) while decays via \(Pr, K^o\) and \(\varepsilon\).
- Mass concentration is in inverse relation with \(\delta, \sigma, K^o, Nt, Nb,\) and \(Sc\).
- Motile density of microorganisms diminished via rising \(\Omega_a, K^o, Lb\) and \(Pe\).
- Magnitude of \(Cf^o\) improves via higher \(K^o, Ha, Fr\) whereas negative impact of \(\Omega\) is recorded.
- Intensity of \(Nu\) is increases for higher \(Pr\) and \(\varepsilon\) while reverse impact of \(Nb, Nt, Ec\) and \(R\) on \(Nu\) is noticed.
- Level of mass transfer rate upsurges for higher estimations of \(\delta, Nt, Nb, n_1, E_1, \sigma\) and \(K^o\) however it retards \(\delta\) against \(E_1\).
- Magnitude of density number is decreasing function of \(\Omega_a, Lb, Pe\) and \(K^o\).

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References